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Non-supersymmetric deformations of non-critical superstrings

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Abstract: We study certain supersymmetry breaking deformations of linear dilaton backgrounds in different dimensions. In some cases, the deformed theory has bulk closed strings tachyons. In other cases there are no bulk tachyons, but there are localized tachyons. The real time condensation of these localized tachyons is described by an exactly solvable worldsheet CFT. We also find some stable, non-supersymmetric backgrounds.

Keywords: Intersecting branes models, Conformal Field Models in String Theory, Brane Dynamics in Gauge Theories, p-branes.

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1. Introduction and summary

In this paper we will study some aspects of string propagation in asymptotically linear dilaton spacetimes. These backgrounds have a boundary at infinity in a spatial direction, ϕ, near which they look like

$$\mathbb{R}^d \times \mathbb{R}_\phi \times \mathcal{N} \,. \tag{1.1}$$

The signature of $\mathbb{R}^d$ is either lorentzian or euclidean. $\mathbb{R}_\phi$ is the real line labelled by ϕ, and $\mathcal{N}$ is a compact space. The dilaton varies with ϕ as follows:

$$\Phi = -\frac{Q}{2}\phi \,, \tag{1.2}$$

where we set $\alpha' = 2$, so the worldsheet central charge of ϕ is $c_\phi = 1 + 3Q^2$. The string coupling $g_s = e^\Phi$ vanishes at the boundary $\phi = \infty$ and grows as we move away from it.

Much of the work on backgrounds of the form (1.1) in recent years focused on solutions that preserve spacetime supersymmetry. The main purpose of this paper is to study non-supersymmetric backgrounds of the form (1.1). We will consider, following [1], solutions of the form

$$\mathbb{R}^d \times \mathbb{R}_\phi \times S^1 \times \frac{\mathcal{M}}{\Gamma}\,, \tag{1.3}$$

where $\mathcal{M}$ is a CFT with $N = 2$ worldsheet supersymmetry, and Γ is a discrete group associated with the chiral GSO projection, which acts on $S^1 \times \mathcal{M}$. When the radius of S^1 is equal to Q, the background (1.3) is spacetime supersymmetric. It was noted in [1] that changing the radius of the S^1 provides a natural way of breaking spacetime supersymmetry continuously (while evading the no-go theorem of [2, 3]). We will study the resulting moduli space of vacua. For $d = 0$ and no $\mathcal{M}$ (the empty theory), this was done in [4]. The main new issue that needs to be addressed for $d > 0$ (or for $d = 0$ and non-trivial $\mathcal{M}$) is the stability of the solution along the moduli space. This will be the focus of our discussion.

There are in fact two kinds of instabilities that can appear in spacetimes of the form (1.1), corresponding to the two kinds of physical states in such spacetimes. One kind is delta function normalizable states in the bulk of the linear dilaton throat $\mathbb{R}_\phi$. These states are characterized by their ϕ momentum, p. Their wavefunctions behave like $\exp(ip\phi)$. The other is normalizable states localized deep inside the throat. Their wavefunctions decay at large ϕ like $\exp(-m\phi)$.

The two kinds of states have a natural interpretation from the spacetime point of view. Backgrounds of the form (1.1), (1.3) typically appear in the vicinity of fivebranes or singularities in string theory [5−8]-[9]. In these geometries, the delta function normalizable states correspond to bulk modes propagating away from the singularity, while the normalizable ones correspond to states localized at the singularity. They are roughly analogous to untwisted and twisted states in a non-compact orbifold. When the theory is supersymmetric these two kinds of states are non-tachyonic. But when we break supersymmetry one or both of them can become tachyonic. We will analyze backgrounds of the form (1.1) in different dimensions, with a particular focus on the question whether they exhibit instabilities of either kind when supersymmetry is broken.

We will see that the stability properties depend on the dimension d and compact manifold $\mathcal{N}$. After discussing some general features in section 2, we turn to examples. In section 3 we consider the case $d = 4$ with $\mathcal{M} = 0$ in (1.3), which corresponds to the near-horizon geometry of the conifold, or equivalently two $NS5$-branes intersecting on an $\mathbb{R}^{3,1}$. We describe the moduli space of non-supersymmetric vacua, and find that everywhere except at the supersymmetric point there is a tachyon in the bulk of the linear dilaton throat.

In sections 4 and 5 we study the case $d = 6$, with $\mathcal{N} = \mathrm{SU}(2)_k$. From a ten dimensional perspective, this is the near-horizon geometry of k parallel $NS5$-branes. The linear dilaton slope (1.2) is given by $Q = \sqrt{2/k}$. In this case changing R corresponds to adding an exactly marginal deformation that breaks the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry associated with the CHS background [5]. We find that if R is not too far from Q, there are no tachyonic bulk modes. The reason for that is that at the supersymmetric point, the bulk modes form a continuum above a gap. Thus, to get tachyons in the bulk one needs to move a finite distance away from the supersymmetric point.

Despite the absence of bulk tachyons, these six dimensional non-supersymmetric backgrounds are unstable, since for all $R \neq Q$ the system has localized tachyons. These have a natural fivebrane interpretation. At the supersymmetric point, there are four flat directions in field space associated with the positions of the fivebranes in the transverse space. These directions are flat since the gravitational attraction of the fivebranes is precisely cancelled by their repulsion due to the NS B-field. Away from the supersymmetric point, the cancellation between the gravitational attraction and the B-field repulsion is spoiled, and the position of the fivebranes in the different directions develops a potential. Two of the directions become massive (i.e. fivebranes separated in these directions attract) while the other two become tachyonic (corresponding to repulsive interactions between the fivebranes). We find the exact worldsheet CFT that corresponds to fivebranes rotating on a circle in the directions in which they attract, and the exact CFT which describes them running away to infinity in the directions in which they repel. In the appendix, we construct the supergravity solutions associated with these CFT's.

In section 6 we consider the case $d = 2$, $\mathcal{M} = 0$ (1.3). As in the $d = 6$ case, there is a finite interval around the supersymmetric point where no bulk tachyons appear. However, in this case there are no localized tachyons either. Thus, this background is an example of a stable, non-supersymmetric deformation of non-critical superstrings. In fact, it is a special case of a more general phenomenon. It was pointed out in [10, 11] that non-critical superstring backgrounds with $Q^2 > 2$ are in a different phase than those with $Q^2 < 2$. While the latter have the property that the high energy spectrum is dominated by non-perturbative states (two dimensional black holes), for the former the high energy spectrum is perturbative [11]. Here we find that there is a difference in the infrared behavior as well. While for $Q^2 < 2$, the lowest lying states for $R \neq Q$ are tachyons that are localized deep in the strong coupling region of the throat, for $Q^2 > 2$ such tachyons are absent. This suggests a kind of UV/IR connection beyond the one familiar in perturbative string theory [12].

The stable, non-supersymmetric models discussed in section 6 all have a linear dilaton slope of order one in string units. In section 7 we consider a three dimensional background, which has the form (1.1), is stable after supersymmetry breaking, but has the property that the slope of the dilaton can be arbitrarily small. This is possible since, as discussed in [13, 14] at the supersymmetric point this model has a finite mass gap.

Other aspects of instabilities in linear dilaton backgrounds were recently considered in [15].

2. Generalities

The $\mathbb{R}_\phi \times S^1$ part of (1.3) is described by two $(1,1)$ worldsheet superfields, Φ and X, whose component form is

$$X = x + \theta\psi_x + \bar{\theta}\bar{\psi}_x + \theta\bar{\theta}F_x \, ,$$

	bulk stability	localized stability	arbitrarily small slope
$d = 4$	$-$	$-$	$-$
$d = 6$	$+$	$-$	$-$
$d = 2$	$+$	$+$	$-$
$d = 3$	$+$	$+$	$+$

Table 1: Stability properties of the different backgrounds considered in the paper.

$$\Phi = \phi + \theta\psi + \bar{\theta}\bar{\psi} + \theta\bar{\theta}F\,. \tag{2.1}$$

The linear dilaton in the ϕ direction, (1.2), implies that the central charge of the two superfields (2.1) is $\hat{c}_L = 2 + 2Q^2$. To form a background of string theory, we add to (2.1) the worldsheet theory corresponding to $\mathbb{R}^{d-1,1}$ and the compact CFT $\mathcal{M}$ such that the total $\hat{c} = 10$. The starting point of our discussion is the spacetime supersymmetric theory of [1]. That theory contains the spacetime supercharges

$$\begin{aligned} Q^+_\alpha &= \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2}} e^{-\frac{i}{2}(H+aZ-Qx)} S_\alpha\,, \\ Q^-_{\bar{\alpha}} &= \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2}} e^{\frac{i}{2}(H+aZ-Qx)} S_{\bar{\alpha}}\,. \end{aligned} \tag{2.2}$$

The notation we use is the following (see [1, 8] for more detail). φ is the bosonized superconformal ghost. S_α and $S_{\bar{\alpha}}$ are spin fields of $\mathrm{Spin}(d-1,1)$. For $d = 2, 6$ they transform in the same spinor representation, while for $d = 4, 8$ they are in different representations. H is a compact scalar field which bosonizes ψ_x, ψ (2.1). The constant a is related to the central charge of the compact CFT $\mathcal{M}$ in (1.3), $a = \sqrt{c_{\mathcal{M}}/3}$. The field Z bosonizes the $\mathrm{U}(1)_R$ current in the $N = 2$ superconformal algebra of $\mathcal{M}$. A similar set of spacetime supercharges arises from the other worldsheet chirality.

The fact that the supercharges (2.2) carry momentum and winding in the x direction means that we should think of x as a compact field. In order for the GSO projection to act as a Z_2, we must take the radius of x to be Q or $2/Q$ (the two are related by T-duality). A natural question is whether increasing and decreasing the radius from the supersymmetric point are equivalent operations. The two are related by changing the sign of the left-moving part of x, and ψ_x, while leaving the right-movers intact. In order to see how this transformation acts on the non-critical superstring, we need to examine its action on the spacetime supercharges (as in [16]). In the notation of equation (2.2), this transformation takes $(H, x) to - (H, x)$, and acts on the compact CFT $\mathcal{M}$ as well, taking $Z \to -Z$. If S_α and $S_{\bar{\alpha}}$ belong to the same spinor representation of $\mathrm{Spin}(d-1,1)$ (which is the case for $d = 2, 6$), this transformation simply exchanges the two lines of (2.2), and we conclude that the supersymmetric theory is self-dual, so that increasing and decreasing R gives rise to isomorphic theories. Otherwise (for $d = 4, 8$), this transformation maps IIA to IIB, which are not isomorphic, like in the critical string [16] (which corresponds to $d = 8$ in our notations).

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A special case of the general construction corresponds to no $\mathcal{M}$ in (1.3). The resulting spacetime is

$$\mathbb{R}^{d-1,1} \times S^1 \times \mathbb{R}_\phi \,, \tag{2.3}$$

and $Q = \sqrt{4 - \frac{d}{2}}$. The geometry (2.3) arises in the vicinity of a singularity of the form

$$z_1^2 + z_2^2 + \cdots + z_{n+1}^2 = 0 \tag{2.4}$$

of a Calabi-Yau n-fold, with $2n + d = 10$ [8]. In particular, the $d = 0$ case discussed in [4] corresponds to a singularity of a Calabi-Yau fivefold. It is believed that (2.3), (2.4) can be alternatively interpreted as the near-horizon geometry of an $NS5$-brane wrapped around the surface

$$z_1^2 + z_2^2 + \cdots + z_{n-1}^2 = 0\,. \tag{2.5}$$

For $d = 6$, (2.5) describes two fivebranes located at the point $z_1 = 0$. For $d = 4$, it describes two fivebranes intersecting along $\mathbb{R}^{3,1}$. For $d = 2, 0$ one has a fivebrane wrapping an A_1 ALE space and a conifold, respectively. The latter can be described in terms of intersecting fivebranes as well, by applying further T-duality in an angular direction of the cone (2.5) .

The lowest lying bulk states that propagate in (2.3) are closed string tachyons, carrying either momentum or winding in the x direction. Their vertex operators have the form

$$e^{-\varphi + ip_L x_L + ip_R x_R + ip_\mu x^\mu + \beta\phi}\,. \tag{2.6}$$

The mass-shell condition is

$$M^2 = \frac{Q^2}{4} - 1 + p_L^2 + \lambda^2 = \frac{Q^2}{4} - 1 + p_R^2 + \lambda^2\,, \tag{2.7}$$

where λ is the momentum in the ϕ direction, $\beta = -\frac{Q}{2} + i\lambda$.

At the supersymmetric point, the GSO projection requires that the left and right moving momenta in the x direction satisfy the constraint $Qp_{L,R} \in 2\mathbb{Z} + 1$. For the momentum modes, $p_L = p_R = \frac{n}{Q}$, and the above constraint implies that $n \in 2\mathbb{Z} + 1$. As we change the radius, one has $p_L = p_R = \frac{n}{R}$ and (2.7) implies that the lowest momentum tachyon (which has $n = 1$) is non-tachyonic when

$$\frac{1}{R^2} \geq 1 - \frac{Q^2}{4} = \frac{d}{8}\,. \tag{2.8}$$

For the winding modes, at the supersymmetric point one has $p_L = -p_R = \frac{wQ}{2}$, and the GSO constraint implies that

$$\frac{1}{2} w Q^2 \in 2\mathbb{Z} + 1\,, \tag{2.9}$$

or

$$w \in \frac{4\mathbb{Z} + 2}{4 - \frac{d}{2}}\,. \tag{2.10}$$

Note that for $d = 2$ the winding w is in general fractional[1]. In the T-dual picture, where the supersymmetric radius is $2/Q$ and the roles of momentum and winding are exchanged, the winding is always integer and the momentum can be fractional.

[1]It is possible to rescale R such that $w \in \mathbb{Z}$.

The condition that the tachyon with the lowest winding, $w = 2/(4-\frac{d}{2})$, be non-tachyonic takes the form

$$R^2 \geq \frac{d}{8} Q^4 \,. \tag{2.11}$$

Combining the conditions (2.8), (2.11) we conclude that for $d = 4$ the theory contains a bulk tachyon for any $R \neq Q$, whereas for $d = 2, 6$ there is a finite range in which the bulk spectrum is non-tachyonic:

$$\begin{aligned} d = 2: &\qquad \frac{3}{2} \leq R \leq 2\,, \\ d = 6: &\qquad \frac{\sqrt{3}}{2} \leq R \leq \frac{2}{\sqrt{3}}\,. \end{aligned} \tag{2.12}$$

In general, the spectrum of the theory in the range (2.12) exhibits a finite gap, which goes to zero as we approach the endpoints.

The line of theories labelled by the radius R which is discussed above is expected, as in the $d = 0$ case of [4], to be part of a richer moduli space of theories which can be obtained by modding out by discrete symmetries. In the next sections we will study the resulting moduli spaces for the different cases, and in particular their stability properties.

An interesting property of these moduli spaces is that when the radius $R \to \infty$, all spacetime fermions are lifted to infinite mass, and one finds the non-critical $0A$ and $0B$ theories. This is different from the situation in the critical and two dimensional string (which correspond in our notations to $d = 8$ and $d = 0$, respectively). The reason is that in the non-compact case, spacetime fermions do not satisfy level matching. Consider, for example, the $(NS, +; R, \pm)$ sector. All states in this sector satisfy

$$L_0 - \bar{L}_0 \in \frac{1}{2} - \frac{d}{16} + \mathbb{Z}\,. \tag{2.13}$$

This is consistent with level matching for $d = 8$ (the critical string) but not for lower dimensions. Similarly, in the $(NS, -; R, \pm)$ sector, one has

$$L_0 - \bar{L}_0 \in -\frac{d}{16} + \mathbb{Z}\,, \tag{2.14}$$

and one can level match in $d = 0$ (the two dimensional string), but not in higher dimensions. Therefore, for $d = 2, 4, 6$ there are no spacetime fermions for infinite radius R, and the theory reduces to type 0.

The bulk modes are described by a free worldsheet theory, and thus can be analyzed using the same methods as in the critical superstring. We will follow the notation of [17] and denote the closed string sectors by

$$\left(\alpha, F, \bar{\alpha}, \bar{F}\right). \tag{2.15}$$

α is the space-time fermion number; it is 0 in the NS sector and 1 in the R sector. F is the world sheet fermion number. In the NS sector, it is 0 in the + sector (e.g. for the graviton) and 1 in the − sector (e.g. for the tachyon). In the Ramond sector, its possible values depend on the dimension. For $d = 4k$, it takes the values 0 and 1, whereas for $d = 2 + 4k$, it takes the values $\pm\frac{1}{2}$. Both α and F are defined modulo two.

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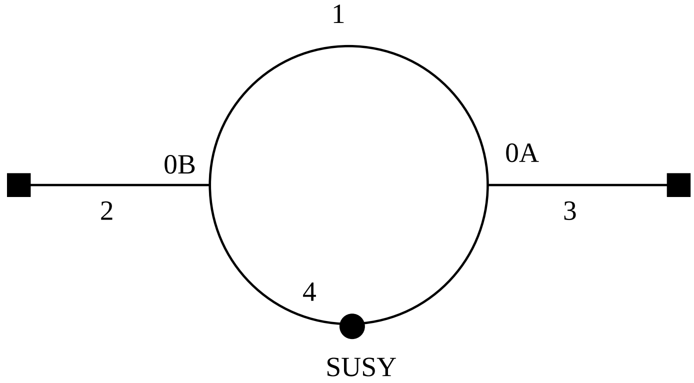

Figure 1: The moduli space of string theories on $\mathbb{R}^{3,1} \times \mathbb{R}_\phi \times S^1$. Line 1 is the usual compactification that connects type 0A to type 0B. Lines 2 and 3 are the super-affine compactifications of 0B and 0A. The black boxes represent the self-dual points along the super-affine lines. The only point at which the theory is free of bulk tachyons is the supersymmetric one which is on the line of twisted compactification (line 4).

3. Four dimensional background

In this section we study string propagation in $\mathbb{R}^{3,1} \times \mathbb{R}_\phi \times S^1$. The dilaton slope (1.2) in this case is $Q = \sqrt{2}$. From a ten dimensional perspective this theory describes the decoupling limit of the conifold [8]. The mutual locality condition is

$$F_1\alpha_2 - F_2\alpha_1 - \bar{F}_1\bar{\alpha}_2 + \bar{F}_2\bar{\alpha}_1 + (\alpha_1\alpha_2 - \bar{\alpha}_1\bar{\alpha}_2) + 2(n_1w_2 + n_2w_1) \in 2\mathbb{Z}\,. \tag{3.1}$$

This condition provides a way to see that in the noncompact case (in which $w_i = 0$), there are no physical states in the (NS, R) and (R, NS) sectors. Indeed, vertex operators in these sectors are not mutually local with respect to themselves.

When we compactify, we find the theories depicted in figure 1. Line 1 describes the compactification of type 0 string theory on a circle. As $R \to \infty$ it approaches the noncompact $0B$ theory; as $R \to 0$ one finds the noncompact $0A$ theory. For any value of R there are bulk tachyons in the spectrum.

Other standard compactifications are the super-affine ones, denoted by lines 2, 3 in figure 1. Since they do not contain spacetime fermions, these compactifications are simple generalizations of the super-affine compactification of critical type 0 string theory on $\mathbb{R}^{8,1} \times S^1$. For example the spectrum of line 2 is

$$\begin{aligned}
&(0,0,0,0)\,, \qquad &&(0,1,0,1)\,, \qquad &&n \in \mathbb{Z}\,, \qquad &&w \in 2\mathbb{Z}\,,\\
&(1,0,1,0)\,, \qquad &&(1,1,1,1)\,, \qquad &&n \in \frac{1}{2} + \mathbb{Z}\,, \qquad &&w \in 2\mathbb{Z}\,,\\
&(1,0,1,1)\,, \qquad &&(1,1,1,0)\,, \qquad &&n \in \mathbb{Z}\,, \qquad &&w \in 2\mathbb{Z}+1\,,\\
&(0,0,0,1)\,, \qquad &&(0,1,0,0)\,, \qquad &&n \in \frac{1}{2} + \mathbb{Z}\,, \qquad &&w \in 2\mathbb{Z}+1\,.
\end{aligned} \tag{3.2}$$

When $R \to \infty$ we get 0B and when $R \to 0$ we get 0A that is equivalent to 0B with $R \to \infty$. Again the spectrum contains tachyons for any value of R.

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Another way to interpolate between the noncompact 0A and 0B theories is to twist by $(-)^{\alpha+F}$. The resulting theory, denoted by line 4 in figure 1, has the spectrum

$$\begin{array}{lllll}
(0,0,0,0)\,, & (1,1,1,1)\,, & n=m\,, & w=l\,, & m-l\in 2\mathbb{Z}\,,\\
(1,0,0,0)\,, & (0,1,1,1)\,, & n=\dfrac{1}{2}+m\,, & w=\dfrac{1}{2}+l\,, & m-l\in 2\mathbb{Z}\,,\\
(0,0,1,0)\,, & (1,1,0,1)\,, & n=\dfrac{1}{2}+m\,, & w=-\dfrac{1}{2}+l\,, & m-l\in 2\mathbb{Z}\,,\\
(0,1,0,1)\,, & (1,0,1,0)\,, & n=1+m\,, & w=l\,, & m-l\in 2\mathbb{Z}\,.
\end{array} \tag{3.3}$$

When $R\to\infty$ we get 0B. When $R\to 0$ we get 0B, which is equivalent to 0A with $R\to\infty$. Thus this line also interpolates between the noncompact 0A and noncompact 0B theories.

Starting with any line in (3.3) we generate the other lines by acting with $(1,0,0,0)(n=\frac{1}{2},w=\frac{1}{2})$ and $(0,0,1,0)(n=\frac{1}{2},w=-\frac{1}{2})$. For $R=Q=\sqrt{2}$ these operators are holomorphic and are identified with the supersymmetry generators (2.2). This is the supersymmetric point of [1]. As mentioned in section 2, this is the only theory in the moduli space depicted in figure 1 which is tachyon free.

4. Six dimensional backgrounds I: stability analysis

A particular linear dilaton spacetime of the form (1.1) is the CHS background [5]

$$\mathbb{R}^{5,1}\times\mathbb{R}_\phi\times \mathrm{SU}(2)_k\,, \tag{4.1}$$

which describes the near-horizon geometry of k $NS5$-branes. The slope of the linear dilaton depends on the number of fivebranes via the relation

$$Q=\sqrt{\frac{2}{k}}\,. \tag{4.2}$$

This background can also be written in the form (1.3), where the compact manifold is $\mathcal{M}_k=\mathrm{SU}(2)_k/\,\mathrm{U}(1)$ (the $N=2$ minimal model with $c=3-\frac{6}{k}$), and the discrete group is $\Gamma=\mathbb{Z}_k$. This follows from the decomposition of $\mathrm{SU}(2)_k$

$$\mathrm{SU}(2)_k=\frac{\left(\frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\times S^1_k\right)}{\mathbb{Z}_k}\,. \tag{4.3}$$

For $k=2$, the $N=2$ minimal model $\mathcal{M}_2$ is empty, and the background (4.1) reduces to

$$\mathbb{R}^{5,1}\times\mathbb{R}_\phi\times S^1\,. \tag{4.4}$$

In the next subsection we study the moduli space corresponding to this case, and then move on to general k.

4.1 Two fivebranes

In this case, F takes the values, $(0,1)$ in the NS sector and $(-\frac{1}{2},\frac{1}{2})$ in the R sector. It is still defined mod 2, however $(-)^F$ belongs to a $\mathbb{Z}_4$ rather than a $\mathbb{Z}_2$ group. The mutual locality condition is

$$F_1\alpha_2 - F_2\alpha_1 - \bar{F}_1\bar{\alpha}_2 + \bar{F}_2\bar{\alpha}_1 - \frac{1}{2}(\alpha_1\alpha_2 - \bar{\alpha_1}\bar{\alpha_2}) + 2(n_1w_2 + n_2w_1) \in 2\mathbb{Z}\,. \tag{4.5}$$

The two noncompact theories are

$$\begin{aligned} 0B: &\qquad (0,0,0,0)\,, \qquad (0,1,0,1)\,, \qquad \left(1,\frac{1}{2},1,\frac{1}{2}\right), \qquad \left(1,-\frac{1}{2},1,-\frac{1}{2}\right), \\ 0A: &\qquad (0,0,0,0)\,, \qquad (0,1,0,1)\,, \qquad \left(1,\frac{1}{2},1,-\frac{1}{2}\right), \qquad \left(1,-\frac{1}{2},1,\frac{1}{2}\right). \end{aligned} \tag{4.6}$$

Upon compactification, we find the theories depicted in figure 2. Lines 1, 2 and 3 are as in the previous section since they do not contain spacetime fermions. Two additional lines of theories (lines 4, 5) are obtained by starting with the supersymmetric theories of [1] (either IIB or IIA) and varying R. At the supersymmetric point, the spectrum is determined by the supercharges, that in the present notation have

$$\begin{aligned} G_1 &= \left(1,\frac{1}{2},0,0\right), \qquad n=\frac{1}{4}\,, \qquad w=\frac{1}{2}\,, \\ G_2 &= \left(0,0,1,\frac{1}{2}\right), \qquad n=\frac{1}{4}\,, \qquad w=-\frac{1}{2}, \end{aligned} \tag{4.7}$$

in type-IIB, and

$$\begin{aligned} G_1 &= \left(1,\frac{1}{2},0,0\right), \qquad n=\frac{1}{4}\,, \qquad w=\frac{1}{2}\,, \\ G_2 &= \left(0,0,1,-\frac{1}{2}\right), \qquad n=\frac{1}{4}\,, \qquad w=-\frac{1}{2}\,, \end{aligned} \tag{4.8}$$

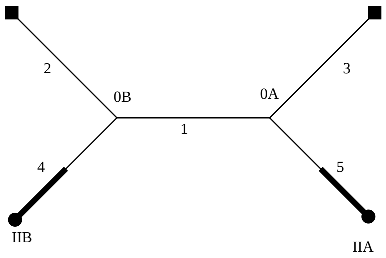

Figure 2: The moduli space of string theory in $\mathbb{R}^{5,1}\times\mathbb{R}_\phi\times S^1$. Lines 1,2 and 3 are as in figure 1. The black boxes represent the self-dual points along the super-affine lines. The thick lines are the regions that are free of bulk tachyons.

in type-IIA. One can check that G_1 and G_2 are mutually local and are holomorphic at $R=Q=1$.

All the states in the theory can be obtained by acting l_1 times with G_1 and l_2 times with G_2 on $(0,0,0,0)$. We denote these states by $[l_1,l_2]=(l_1,\frac{l_1}{2},l_2,\pm\frac{l_2}{2})$, with a plus sign in type-IIB and a minus sign in type-IIA. α and F are defined modulo two, thus $0\le l_1,l_2\le 3$. The momentum and winding modes in each of the sixteen sectors are determined by the mutual locality of $[l_1,l_2]$ with respect to G_1 and G_2. From (4.5) we get

$$n\in\mathbb{Z}+\frac{1}{4}(l_1+l_2)\,,\ \ w\in 2\mathbb{Z}+\frac{1}{2}(l_1-l_2)\,,\ \ n+\frac{w}{2}\in 2\mathbb{Z}+\frac{l_1}{2}\,. \tag{4.9}$$

The tachyon field, for example, is in the $[2,2]=(0,1,0,1)$ sector. Thus the allowed winding and momentum modes for the tachyon satisfy

$$n,\frac{w}{2}\in\mathbb{Z}\,, \qquad n+\frac{w}{2}\in 2\mathbb{Z}+1\,. \tag{4.10}$$

The lowest momentum mode has $n = 1$, while the lowest winding mode has $w = 2$, in agreement with the discussion of section 2.

Since the spectrum is presented above in terms of n and w (as opposed to p_L and p_R), it is simple to generalize the discussion to arbitrary radius. Away from the supersymmetric point G_1 and G_2 are no longer holomorphic; hence, SUSY is broken. As explained in section 2, the theory is tachyon free for

$$\frac{3}{4} \leq R^2 \leq \frac{4}{3}\,. \tag{4.11}$$

The supersymmetric point, $R = 1$, is in this range. As explained in section 2, it is self-dual; it is clear from (4.1), that at this point there is an enhanced $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry. As $R \to 0, \infty$ we find the noncompact 0B, 0A theories.

4.2 k fivebranes

In this subsection we discuss the CHS background [5] (4.1) in the presence of the supersymmetry breaking deformation

$$\lambda \int d^2z J_3 \bar{J}_3\,, \tag{4.12}$$

where $J_3, \bar{J}_3$ are particular Cartan subalgebra generators of $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$. Such a deformation has been investigated by many authors including [18 − 20]-[21]. Using the decomposition (4.3), this corresponds to changing the radius of the S^1 away from its original value. From the spacetime point of view, this deformation squashes the three-sphere transverse to the fivebranes, and breaks the SO(4) symmetry of rotations around the fivebranes down to $\mathrm{SO}(2) times\,\mathrm{SO}(2)$.

The lightest modes in the background (4.1) are gravitons, whose vertex operators are given, in the $(-1,-1)$ picture, by[2]

$$\left(\psi\bar{\psi}V_j\right)_{j+1;m,\bar{m}}\,. \tag{4.13}$$

The dimension of these operators is

$$\Delta = \frac{1}{2} + \frac{j(j+1)}{k}\,, \tag{4.14}$$

where $j = 0, \frac{1}{2}, 1, \cdots, \frac{k}{2} - 1$. From the point of view of the decomposition (4.3) we can think of (4.14) as being a sum of two contributions. One comes from the $\frac{\mathrm{SU}(2)}{\mathrm{U}(1)}$ part and is equal to

$$\Delta^{\frac{\mathrm{SU}(2)}{\mathrm{U}(1)}} = \frac{1}{2} + \frac{j(j+1) - m^2}{k}\,, \qquad m = -j-1, -j, -j+1, \cdots, j+1\,, \tag{4.15}$$

with a similar formula for the right-movers. The second contribution to the dimension comes from the U(1) part and is equal to

$$\Delta^{\mathrm{U}(1)} = \frac{m^2}{k}\,, \tag{4.16}$$

[2]To simplify the equations we have omitted the contribution of $\mathbb{R}^{5,1} \times \mathbb{R}_\phi$ to these vertex operators.

again with a similar formula for the right-movers. The U(1) part of the dimension comes from momentum and winding on a circle. Thus, we have

$$\Delta^{\mathrm{U}(1)} = \frac{1}{2}p_L^2\,, \qquad \bar{\Delta}^{\mathrm{U}(1)} = \frac{1}{2}p_R^2\,, \tag{4.17}$$

with

$$p_L = \frac{n}{R} + \frac{wR}{2}\,, \qquad p_R = \frac{n}{R} - \frac{wR}{2}\,. \tag{4.18}$$

Working in conventions where the winding number is an integer, we have

$$\begin{aligned} R &= Q = \sqrt{\frac{2}{k}}\,, \\ n &= \frac{m+\bar{m}}{k}\,, \\ w &= m - \bar{m}\,. \end{aligned} \tag{4.19}$$

Note that while the winding number is an integer, the momentum can be fractional, $n \in \mathbb{Z}/k$ due to the $\mathbb{Z}_k$ orbifold in (4.3). States that carry non-trivial $\mathbb{Z}_k$ charge in $\frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}$ have fractional momentum n (4.19).

Now we change the radius from the supersymmetric point $R_{\mathrm{susy}} = Q = \sqrt{\frac{2}{k}}$ to some other value, R. We would like to find the values of R for which the theory is tachyon free. As we vary R, the operators (4.13) retain the property that $L_0 = \bar{L}_0$, so it is sufficient to focus on, say, the left moving part. The dimension of the operator (4.13) for generic R is given by (omitting the $\frac{1}{2}$ in (4.15)):

$$\frac{j(j+1) - m^2}{k} + \frac{1}{2}p_L^2\,. \tag{4.20}$$

Adding the Liouville contribution, we have

$$\frac{(j+\frac{1}{2})^2 - m^2}{k} + \frac{1}{2}p_L^2 - \frac{1}{2}\left(\beta + \frac{Q}{2}\right)^2\,. \tag{4.21}$$

The delta-function normalizable states in the cigar have $\beta = -\frac{Q}{2} + ip$, so the last term in (4.21) is positive. If the expression (4.21) is positive, the relevant state is massive, while if it is negative, it is tachyonic.

We see immediately that for all $|m| \leq j$, the states in the throat are never tachyonic. The only way to get tachyons for any radius R is to consider the case $|m| = |\bar{m}| = j+1$, i.e. $m = \bar{m} = j+1$, or $m = -\bar{m} = j+1$. The two are related by T duality, so it is enough to analyze the first, which corresponds to pure momentum modes, with $p_L = p_R = \frac{2(j+1)}{kR}$. The condition for this momentum mode to be non-tachyonic is

$$\frac{(j+\frac{1}{2})^2 - (j+1)^2}{k} + \frac{1}{2}p_L^2 > 0 \tag{4.22}$$

or

$$R^2 < \frac{2}{k}\frac{(j+1)^2}{j+\frac{3}{4}}\,. \tag{4.23}$$

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Thus $j = 0$ is the first state that becomes tachyonic. This happens at $R^2 = \frac{8}{3k}$, i.e. $\frac{4}{3}$ times the original radius at the supersymmetric point. Repeating the analysis for winding modes we conclude that the theory is (bulk) tachyon free for

$$\frac{3}{2k} \le R^2 \le \frac{8}{3k}\,, \tag{4.24}$$

in agreement with the result we got for $k = 2$, (4.11).

4.3 Localized instabilities

In the previous subsections we saw that in the six dimensional case, there is a finite range of radii, for which the bulk theory in the fivebrane throat is non-tachyonic (in fact, it exhibits a finite mass gap). This is different from the four dimensional case, where the gap vanishes, and bulk tachyons appear immediately as we change the radius. We note in passing that in other four dimensional linear dilaton backgrounds, where the manifold $\mathcal{M}$ in (1.3) is non-trivial, the gap is finite as well, and the situation is similar to the six dimensional examples.

In theories with a finite gap in the bulk spectrum, there are no bulk instabilities even when we break supersymmetry (sufficiently mildly). A natural question, which we will address in this subsection, is whether such theories have localized instabilities.

In the six dimensional system (4.1) we understand the low energy dynamics in terms of the theory on $NS5$-branes. Consider, for example, the type-IIB case.[3] At the supersymmetric point, the low energy theory on the fivebranes is a six dimensional super Yang-Mills theory with sixteen supercharges, and gauge group SU(k) (for k fivebranes). This theory has a moduli space parameterized by the locations of the fivebranes in the transverse $\mathbb{R}^4$. Denoting the positions of the fivebranes in the four transverse directions by the $k \times k$ matrices

$$A = x_6 + ix_7\,, \qquad B = x_8 + ix_9\,, \tag{4.25}$$

one can parameterize points in the moduli space of separated fivebranes by the expectation values of gauge invariant operators such as $\mathrm{Tr}A^n$, $\mathrm{Tr}B^n$, etc.

In the supersymmetric theory, it is well understood [7, 22, 23] how to describe separated fivebranes in the geometry (4.1). For example, if the fivebrane configuration has a non-zero expectation value of $\mathrm{Tr}B^{2j+2}$, the worldsheet theory should be deformed by the operator

$$e^{-\varphi-\bar\varphi}\psi^+\bar\psi^+V_{j;j,j}e^{-Q(j+1)\phi}\,. \tag{4.26}$$

This operator corresponds to a wavefunction that is supported in the strong coupling region of the linear dilaton space $\mathbb{R}_\phi$. It is an example of a localized state in the throat. This is in agreement with the spacetime picture, according to which it describes a scalar living on the fivebranes.

A particularly symmetric deformation of the kind (4.26) that has been extensively studied corresponds to placing the fivebranes on a circle of size r_0 in the B plane

$$B_l = r_0 e^{\frac{2\pi i l}{k}}\,, \qquad A_l = 0\,, \qquad l = 1, 2, \ldots, k\,. \tag{4.27}$$

[3] A similar discussion holds for IIA fivebranes.

In the bulk, this is described by condensation of the mode (4.26) with $j = \frac{k}{2} - 1$. It is useful to write this operator in the decomposition (4.3). By looking at equation (4.15), and using the fact that it has $m = j + 1 = \frac{k}{2}$, we see that from the point of view of $\frac{\mathrm{SU}(2)}{\mathrm{U}(1)}$ it is proportional to the identity operator. In fact, it is the $N = 2$ Liouville superpotential

$$W = e^{i\frac{1}{R}x - \frac{1}{Q}\phi}, \tag{4.28}$$

where $R = Q$ is the radius at the supersymmetric point (4.19). As is well known, the $N = 2$ Liouville operator is truly marginal, and signals the change from $\mathbb{R}_\phi \times S^1$ to the cigar (here we are describing it in terms of the T-dual variables, so the N=2 Liouville perturbation is a momentum mode). Note that we are not assuming here that the string coupling is weak everywhere. It could be that the string coupling at the tip of the cigar is very large, but we are studying the weakly coupled region where it is small.

Now suppose we change the radius R from its original value (4.19). There are two different cases to discuss. If we make R smaller, the operator (4.28), which had dimension one before, will have dimension larger than one. This means that it is massive, and we have to dress (4.28) with a time-dependent part, like $\cos(Et)$ with a real energy E. This means that $\langle \mathrm{Tr} B^k \rangle$, and the size of the circle on which the fivebranes are placed, r_0 (4.27), will oscillate between two extreme values.

On the other hand, if we increase the radius R, the dimension of (4.28) drops below one, it becomes tachyonic, and to make it physical we have to dress it with, e.g. , $\cosh(Et)$ with a real E. This means that r_0 increases without bound (in this approximation) as $|t| \to \infty$. In the next section we explore in details these dynamical processes using various techniques.

5. Six dimensional backgrounds II: time dependence, field theory and gravity

In this section we further discuss the deformed $NS5$-brane background with broken supersymmetry obtained by applying the perturbation (4.12). In the first subsection we construct time-dependent solutions that correspond to rolling fivebranes. In the second subsection we comment on the description of the deformed fivebrane system using the low energy field theory and supergravity. The latter is valid at large k.

5.1 Rolling fivebranes

At the end of the last section we briefly mentioned the time-dependent solutions obtained by displacing the fivebranes from the origin of the moduli space $A = B = 0$ (4.25) in the presence of the supersymmetry breaking deformation (4.12). We only discussed what happens to leading order in the separation of the fivebranes (4.28), since we only imposed the leading order condition that the dressed perturbation be marginal. In general, one expects the solution to receive corrections, and for the solutions mentioned in the previous section we expect these to be large. The reason is that these solutions correspond to accelerating fivebranes, and are expected to radiate. Since the tension of the $NS5$-branes scales like $1/g_s^2$, the closed strings radiation (that goes like $G_N T_5$) is an order one effect that influences the solution in a non-trivial way at tree level.[4]

Nevertheless, it is possible to find exact solutions of the classical string equations of motion that correspond to rolling fivebranes. Consider first the case of decreasing the radius R, such that the $N=2$ Liouville operator (4.28) becomes irrelevant (or massive). In that case, we can make it marginal by replacing (4.28) with

$$W = e^{\frac{i}{R}x + i\omega_R t - \frac{1}{Q}\phi}\,. \tag{5.1}$$

The mass-shell condition requires that

$$\omega_R^2 = \frac{1}{R^2} - \frac{1}{Q^2}\,. \tag{5.2}$$

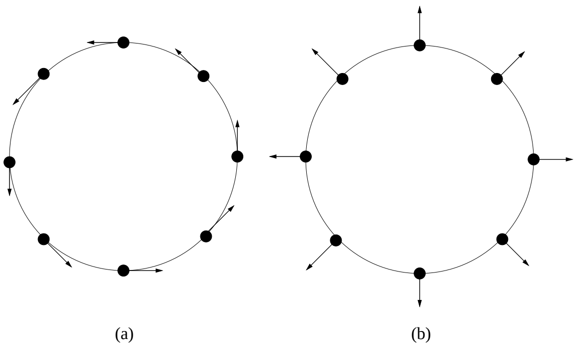

Figure 3: Time dependent processes which have exact CFT descriptions: (a) Fivebranes rotating around a circle with a constant angular velocity; (b) Fivebranes moving radially with a constant radial velocity.

We can define a coordinate $x_{\rm new}$ by

$$\frac{1}{Q}x_{\rm new} = \frac{1}{R}x + \omega_R t\,. \tag{5.3}$$

The condition (5.2) implies that $x_{\rm new}$ is canonically normalized (when x and t are). In terms of this new field, the perturbation separating the fivebranes (5.1) looks precisely like the $N=2$ Liouville perturbation again. Thus, we conclude that (5.1) is a truly marginal deformation. It describes the fivebranes placed on the slope of the quadratic potential and rotating around the circle with angular velocity ω_R (see figure 3a). The existence of an exact CFT description implies that there should also be a supergravity solution associated with that system. This solution is described in the appendix.

The orthogonal combination of (t,x) is timelike,

$$\frac{1}{Q}t_{\rm new} = \frac{1}{R}t + \omega_R x\,. \tag{5.4}$$

The original periodicity of x, $x \sim x + 2\pi R$ translates in terms of the new coordinates to:

$$(t_{\rm new}, x_{\rm new}) \sim (t_{\rm new}, x_{\rm new}) + 2\pi Q(\omega_R R, 1)\,. \tag{5.5}$$

One can describe the solution of fivebranes rotating around the circle by a coset CFT. The supersymmetric solution corresponding to fivebranes on a circle can be thought of as

$$\mathbb{R}^{5,1} \times {\rm SL}(2,\mathbb{R})_k \times {\rm SU}(2)_k \tag{5.6}$$

modded out by the null U(1) symmetry generated by the current

$$J = J_3 - K_3\,, \tag{5.7}$$

[4]This is different from the well studied case of unstable D-branes with a rolling tachyon [24]. There, the closed strings radiation is an order g_s effect [25, 26], and so can be neglected (at least formally) in the leading approximation.

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where J_3 and K_3 are the Cartan subalgebra generators of SL(2) and SU(2) respectively. In order to go from this solution to the one described by (5.1), we need to mix the U(1) current K_3 with the timelike U(1), $i\partial t$. Ignoring the periodicity conditions, this is simply a boost transformation. Thus, in terms of the coset description (5.6), (5.7), we want to gauge the current

$$J_3 - \beta_1 K_3 - \frac{i\beta_2}{Q}\partial t\,, \tag{5.8}$$

where the boost parameters β_i satisfy $\beta_1^2 - \beta_2^2 = 1$. This provides an exact CFT description of the system of fivebranes rotating around the circle.

Naively, one might expect that as the fivebranes are rotating around the circle they should radiate, and the solution should be modified, already at tree level. This seems in contradiction to the fact that we found an exact solution of the full equations of motion of classical closed string theory. The resolution of this puzzle is that in the near-horizon limit, the solution with fivebranes rotating around the circle corresponds to motion with constant velocity, and not constant acceleration (since the near-horizon metric for the angular S^3 is $d\Omega^2$ and not $r^2 d\Omega^2$). Thus, it does not radiate.

This should be contrasted with the case where we place the fivebranes at some distance from the origin and let them go. In this case, one expects the system to oscillate about the minimum, with $r_0 \sim \cos\omega_R t$. This amounts to replacing $\exp(i\omega_R t)$ in (5.1) with $\cos(\omega_R t)$. From the spacetime point of view, we expect non-zero radiation in this case, since this solution corresponds to a time dependence of ϕ of the form $\phi \sim \log\cos\omega_R t$. Thus, there is in this case non-zero acceleration, and we expect the solution to be modified at the tree level. This is natural from the worldsheet point of view. If we replace $\exp(i\omega_R t)$ by $\cos(\omega_R t)$ in (5.1), the operator (5.1) ceases to be holomorphic, and one expects it to become marginally relevant (like the marginal Sine-Gordon and non-abelian Thirring model couplings). Presumably, this behavior of the worldsheet theory is directly related to the classical radiation that one expects in spacetime.

For the other case, of increasing[5] radius R, for which w_R is imaginary, the operator (4.28) is relevant, and we expect to have to dress it with a real exponential in time. Thus, instead of (5.1) we now have

$$W = e^{\frac{i}{R}x + w_R t - \frac{1}{Q}\phi}\,, \tag{5.9}$$

and w_R is given by (similarly to (5.2))

$$w_R^2 = \frac{1}{Q^2} - \frac{1}{R^2}\,. \tag{5.10}$$

We would like again to make an $N = 2$ Liouville perturbation out of (5.9), so we define a new Liouville coordinate $\phi_{\rm new}$ by

$$\frac{1}{R}\phi_{\rm new} = \frac{1}{Q}\phi - w_R t\,. \tag{5.11}$$

[5] T-duality relates increasing R to decreasing R, while exchanging momentum and winding modes. Therefore, instead of increasing R we can continue to decrease it and study other classical solutions.

Again, due to (5.10), $\phi_{\rm new}$ is canonically normalized. Now, in terms of $\phi_{\rm new}$ and x we again have an $N = 2$ Liouville theory, so this is an exact solution of the equations of motion of the theory, corresponding to fivebranes approaching the origin of the B plane as $t \to -\infty$, and moving out to infinity at $t \to +\infty$ (see figure 3b). This is an analog of the "half-brane" from unstable D-brane dynamics. Note that, as in the discussion of fivebranes rotating around a circle, during the time evolution no radiation is emitted, since the fivebranes are moving with constant velocity in the ϕ direction. The relevant "rolling fivebrane" supergravity solution is constructed in the appendix.

One thing that is different in this case relative to the previous one is that since we are mixing the time coordinate with ϕ to make the cigar, the remaining combination of ϕ and t,

$$\frac{1}{R} t_{\rm new} = \frac{1}{Q} t - \omega_R \phi \tag{5.12}$$

which does not participate in the cigar and is timelike, has a linear dilaton associated with it. More precisely, one finds,

$$Q_{\phi_{\rm new}} = R\,, \qquad Q_{t_{\rm new}} = Q R \omega_R\,. \tag{5.13}$$

This signals the fact that at early times ($t, t_{\rm new} \to -\infty$), when the fivebranes are on top of each other, we have a strong coupling problem, despite the fact that we have constructed a cigar which avoids the strong coupling problem in the direction $\phi_{\rm new}$.

Again, one can describe the time-dependent background (5.9) by an exact coset CFT. As we see from (5.11), (5.12), in this case we need to perform a boost mixing ϕ and t, or J_3 and ∂t. One can describe this background as (5.6) modded out by the null current

$$J_3 + \frac{\beta_1}{Q} \partial t - \beta_2 K_3\,, \tag{5.14}$$

with $\beta_1^2 + \beta_2^2 = 1$.

The solution describing fivebranes running to infinity as a function of time is an example of localized tachyon condensation. In our case we are able to find an exact CFT that describes this time-dependent process, and smoothing out of the singularity, while in the case of non-supersymmetric orbifolds [27] no exact time-dependent solutions of this sort were found. The reason for the difference is that we took the near-horizon limit of a throat, in which, as we have seen, the smoothing out of the singularity proceeds via a process in which no radiation is emitted, whereas the analysis of [27], takes place in the full geometry where one expects the solution to be much more complicated and to involve non-trivial radiation effects.[6]

A related point is that here we wrote an exact solution which is an analog of a half S-brane [29, 24] for decaying D-branes. There should exist a more complicated solution where the distance between the fivebranes starts at $t \to -\infty$ very large, decreases to a minimal value, and then increases back to infinity. This solution would be an analog of the full S-brane. In this case the ϕ coordinate accelerates near the turning point, and one expects non-trivial radiation effects at tree level.

[6] The relation between localized tachyons on non-supersymmetric orbifolds and throat theories was discussed in [28].

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5.2 Dual field theory and supergravity

In this subsection we consider the deformation (4.12) from the point of view of the dual field theory and supergravity descriptions. We focus on the type-IIB case. In this case the dual field theory is a six dimensional SYM theory with sixteen superchrages and an SU(k) gauge group. This field theory is infrared free and is believed to resolve the strong coupling singularity associated with the CHS background [6]. Since the wave function of (4.26) is concentrated in the strong coupling region, one can study the relevant dynamics using the dual field theory description.

The relationship between the bulk modes in the CHS background and chiral field theory operators was described in [7]. The vertex operators (4.13) correspond to symmetric, traceless combinations of the four scalars (4.25). In particular, the deformation (4.12) is dual to a mass term for the scalars

$$\alpha\,\mathrm{Tr}(A^*A - B^*B)\,. \tag{5.15}$$

Thus, with the deformation two of the four flat directions become massive, while the other two become tachyonic. This fits nicely with the results of the previous subsection. The relation between α and R is the following. Normalizing the field B such that the lagrangian takes the form

$$\mathcal{L} = |\dot{B}|^2 + \alpha|B|^2\,, \tag{5.16}$$

and comparing the solution of the equation of motion $B = r_0 e^{\pm i\sqrt{\alpha}t}$ to (5.1), we find that

$$\alpha = \frac{1}{Q^2} - \frac{1}{R^2}\,. \tag{5.17}$$

Another way to study the dynamics is to use the supergravity description, which is accurate for large k. First we have to find the deformation of the CHS background associated with (4.12). In the appendix we describe the relevant solution from the coset point of view. Here we describe this solution directly in the CHS language.

The starting point is the CHS background associated with k $NS5$-branes [5],

$$\begin{aligned} ds^2 &= dx_{||}^2 + d\phi^2 + 2k\left(d\theta^2 + \sin^2\theta d\phi_1^2 + \cos^2\theta d\phi_2^2\right), \\ B &= k(1+\cos(2\theta))d\phi_1\wedge d\phi_2\,, \qquad g_s^2 = \exp\left(-\sqrt{\frac{2}{k}}\phi\right), \end{aligned} \tag{5.18}$$

where $0\le\theta\le\pi/2,\ 0\le\phi_1,\ \phi_2\le 2\pi$. The transverse coordinates (4.25) are given by

$$A = \sqrt{2k}\exp\left(\phi\sqrt{\frac{1}{2k}}\right)\sin(\theta)\exp(i\phi_1)\,, \qquad B = \sqrt{2k}\exp\left(\phi\sqrt{\frac{1}{2k}}\right)\cos(\theta)\exp(i\phi_2)\,. \tag{5.19}$$

This relation can be derived by comparing the field theory and the supergravity expressions for the energy of a BPS $D1$-brane (or of a fundamental string in the near-horizon geometry of k $D5$-branes).

To find the solution associated with the perturbation (4.12) we recall that this deformation can be viewed [30, 18, 31] as an SL(2, $\mathbb{R}$) transformation of the usual CHS background. The relevant SL(2, $\mathbb{R}$) transformation is

$$\tau \to \tau' = \frac{\tau}{1+\alpha\tau}\,, \tag{5.20}$$

where τ is the Kahler parameter, $\tau \equiv B_{12} + i\sqrt{g} = 2k\cos(\theta)e^{i\theta}$. A short calculation yields

$$\begin{aligned} ds^2 &= dx_{||}^2 + d\phi^2 + 2k\left(d\theta^2 + \frac{1}{(1+k\alpha)^2+\tan^2\theta}(\tan^2\theta d\phi_1^2 + d\phi_2^2)\right), \\ B &= \frac{2k(1+k\alpha)}{(1+k\alpha)^2+\tan^2\theta}d\phi_1\wedge d\phi_2\,, \qquad g_s^2 = e^{-\phi\sqrt{2/k}}\frac{1+\tan^2\theta}{(1+k\alpha)^2+\tan^2\theta}\,. \end{aligned} \tag{5.21}$$

This solution is not quite what we are after since it has a conical singularity at $\theta = 0$. This can be fixed by rescaling $\phi_1 \to \phi_1/(1+k\alpha)$, which yields

$$\begin{aligned} ds^2 &= dx_{||}^2 + d\phi^2 + 2k\left(d\theta^2 + \frac{1}{L^2+\tan^2\theta}(L^2\tan^2\theta d\phi_1^2 + d\phi_2^2)\right), \\ B &= \frac{2kL^2}{L^2+\tan^2\theta}d\phi_1\wedge d\phi_2\,, \qquad g_s^2 = e^{-\phi\sqrt{2/k}}\frac{1+\tan^2\theta}{L^2+\tan^2\theta}\,, \end{aligned} \tag{5.22}$$

where $L = 1+\alpha k$. The curvature in string units is small as long as $k \gg 1$ and $L-1 \ll 1$.

We wish to show that this background indeed exhibits the same physics as (5.15). Namely, it has two massive directions and two tachyonic ones. To this end we calculate the potential felt by a probe fivebrane that is localized in ϕ and on the sphere. This potential is the strong coupling dual of (5.15). In general such a calculation need not yield the same result. However, since for $|\alpha| \ll 1$ we are in the near BPS limit we expect to find the same potential to leading order in α.

To see that this is indeed the case, consider the DBI action of a probe $D5$-brane propagating in the S-dual background,

$$\begin{aligned} ds^2 &= g_s\left[dx_{||}^2 + d\phi^2 + 2k\left(d\theta^2 + \frac{1}{L^2+\tan^2\theta}(L^2\tan^2\theta d\phi_1^2 + d\phi_2^2)\right)\right], \\ F_3 &= \frac{-4kL^2\tan(\theta)(1+\tan^2\theta)}{(L^2+\tan^2\theta)^2}d\phi_1\wedge d\phi_2\wedge d\theta\,, \qquad g_s^2 = e^{\phi\sqrt{2/k}}\frac{L^2+\tan^2\theta}{1+\tan^2\theta}\,. \end{aligned} \tag{5.23}$$

The six-form potential that couples to the $D5$-brane probe is $A_{||} = Le^{\phi\sqrt{2/k}}$ and the DBI potential reads

$$V = g_s^{-1}\sqrt{g_{||}} - A_{||} = e^{\phi\sqrt{2/k}}\left(\frac{L^2+\tan^2\theta}{1+\tan^2\theta} - L\right). \tag{5.24}$$

Using (5.19) we can express this potential in terms of the field theory variables. For small deformation, $\alpha \ll 1$, we find,

$$V = \alpha\left(A^*A - B^*B\right)\,, \tag{5.25}$$

in agreement with (5.15) .

6. Two dimensional background

In this section we study superstrings on $\mathbb{R}^{1,1} \times \mathbb{R}_\phi \times S^1$, with $Q = \sqrt{3}$. The analysis here is fairly similar to the one in section 4.1. Like in the six dimensional case, F takes the values $(0, 1)$ in the NS sector and $(-\frac{1}{2}, \frac{1}{2})$ in the R sector. The mutual locality condition is slightly different than in the six dimensional case

$$F_1\alpha_2 - F_2\alpha_1 - \bar{F}_1\bar{\alpha}_2 + \bar{F}_2\bar{\alpha}_1 + \frac{1}{2}(\alpha_1\alpha_2 - \bar{\alpha_1}\bar{\alpha_2}) + 2(n_1 w_2 + n_2 w_1) \in 2\mathbb{Z}\,. \tag{6.1}$$

The moduli space is very similar to that of figure 2. The type 0 and super-affine lines $1, 2, 3$ are as there. As in the six dimensional case, there are two additional lines of theories that are obtained by starting with the supersymmetric theories (either IIA or IIB) and varying R. These theories can be analyzed using the same approach as in section 4.1. Now the supersymmetry generators are

$$\begin{aligned} G_1 &= \left(1, \frac{1}{2}, 0, 0\right), \qquad n = \frac{3}{4}, \qquad w = \frac{1}{2}, \\ G_2 &= \left(0, 0, 1, \frac{1}{2}\right), \qquad n = \frac{3}{4}, \qquad w = -\frac{1}{2}, \end{aligned} \tag{6.2}$$

in type-IIB, and

$$\begin{aligned} G_1 &= \left(1, \frac{1}{2}, 0, 0\right), \qquad n = \frac{3}{4}, \qquad w = \frac{1}{2}, \\ G_2 &= \left(0, 0, 1, -\frac{1}{2}\right), \qquad n = \frac{3}{4}, \qquad w = -\frac{1}{2}, \end{aligned} \tag{6.3}$$

in type-IIA. With the help of (6.1) we find that the winding and momentum modes in the $[l_1, l_2]$ sector satisfy

$$n \in \mathbb{Z} - \frac{1}{4}(l_1 + l_2)\,, \qquad 3w \in 2\mathbb{Z} + \frac{1}{2}(l_2 - l_1)\,, \qquad n + \frac{3}{2}w \in 2\mathbb{Z} - \frac{1}{2}l_1\,. \tag{6.4}$$

For the tachyon modes (in the $[2, 2]$ sector), we find, in agreement with section 2, that the lowest momentum is one, and the lowest winding is $\frac{2}{3}$. As explained in section 2 this implies that the theory is free of bulk tachyons for $\frac{3}{2} \le R \le 2$. The supersymmetric radius, $R = \sqrt{3}$, is self-dual under T-duality. However, here, unlike the six dimensional case, the symmetry is not enhanced to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ at this point. Again, these lines interpolate between 0B (0A) and 0B (0A).

While the physics in the bulk of the linear dilaton throat is quite similar to the six dimensional case, the localized dynamics is different. In the six dimensional case, there was an instability due to localized tachyons such as the $N = 2$ Liouville mode (4.28), which was massless for $R = Q$ and became tachyonic when $R \neq Q$. This deformation exists also in the two dimensional case. However since now $Q > \sqrt{2}$, this mode is non-normalizable (for recent discussions, see [10, 11]), and hence it cannot dynamically condense. In fact, in this case there are *no* normalizable localized tachyons, so the non-supersymmetric model is locally stable.

7. Three dimensional backgrounds

The theories we considered in the previous sections have the property that the $N = 2$ Liouville operator (4.28) is in the spectrum. Since this mode is tachyonic for $R \neq Q$, $Q < \sqrt{2}$, one might be tempted to conclude that there are no stable non-supersymmetric deformations of linear dilaton backgrounds with a small dilaton slope. This conclusion is incorrect.

A counter example is the background

$$\mathbb{R}^{2,1} \times \mathbb{R}_\phi \times \mathrm{SU}(2)_{k_1} \times \mathrm{SU}(2)_{k_2}\,, \tag{7.1}$$

with

$$Q = \sqrt{\frac{2}{k}}\,, \qquad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}\,. \tag{7.2}$$

We will see that a small non-supersymmetric deformation of the form (4.12), acting on either of the two SU(2)'s in (7.1), does not lead to instabilities in this case.

The background (7.1) is the near-horizon geometry of the intersection of k_1 coincident $NS5$-branes stretched in the directions (012345) and k_2 coincident $NS5$-branes stretched in (016789) (see [13] for a recent discussion). This brane configuration is Poincare invariant in 1+1 dimensions, but the near-horizon geometry (7.1) exhibits a higher Poincare symmetry, in $2 + 1$ dimensions [13] .

In section 4 we considered the system of k parallel $NS5$-branes and argued in three different ways for instability when (4.12) is turned on. For the background (7.1) all these arguments indicate that the system is stable.

(1) From the worldsheet point of view the instability was due to the fact that the $N = 2$ Liouville mode became tachyonic when we changed the radius. Now, however, this mode is not in the spectrum [13] . This is possible because this theory is not of the form (1.3).

(2) The dual field theory argument for the instability was that some of the flat directions at $R = R_{\rm susy}$ became tachyonic when $R \neq R_{\rm susy}$. The field theory dual to (7.1) , however, has a mass gap [13, 14]. Hence a small deformation of the parameters cannot lead to a tachyonic mode. The flat directions in the case of k parallel $NS5$-branes were associated with moving the branes in the transverse directions. In our case these deformations are massive, since each stack of fivebranes is wrapped around a three-sphere (see [13] for a more extensive discussion).

(3) A complementary argument for instability came from the probe fivebrane dynamics (see section 5.2). Is it possible that a probe fivebrane in the background (7.1) that respects three dimensional Poincare invariance has a flat direction and can become unstable away from the supersymmetric point? To verify that this is not the case let us calculate the potential experienced by such a probe fivebrane. Again it is convenient to work with S-dual variables and study the DBI action for a probe $D5$-brane. The S-dual metric and dilaton take the form

$$ds^2 = g_s\left[-dx_0^2 + dx_1^2 + dx_2^2 + d\phi^2 + 2k_1 d\Omega_3^2 + 2k_2 d\tilde{\Omega}_3^2\right], \qquad g_s^2 = e^{\phi Q}\,, \tag{7.3}$$

and the RR-fields are

$$F_3 = 2k_1 \sin(2\theta) d\phi_1 \wedge d\phi_2 \wedge d\theta + 2k_2 \sin(2\tilde{\theta}) d\tilde{\phi}_1 \wedge d\tilde{\phi}_2 \wedge d\tilde{\theta}\,. \tag{7.4}$$

To compute the DBI action of a probe $D5$-brane we need the dual field strength

$$\begin{aligned} F_7 &= \star F_3 \\ &= 2e^{Q\phi}\left[k_1 \sin(2\tilde{\theta})\left(\frac{k_2}{k_1}\right)^{3/2} dx_{||}\wedge d\phi \wedge d\tilde{\Omega} + k_2 \sin(2\theta)\left(\frac{k_1}{k_2}\right)^{3/2} dx_{||}\wedge d\phi \wedge d\Omega\right], \end{aligned} \tag{7.5}$$

from which we find the six-form that couples to the D5-brane

$$A_6 = 2e^{Q\phi}\left[\frac{k_1}{Q}\left(\frac{k_2}{k_1}\right)^{3/2} \sin(2\tilde{\theta}) dx_{||}\wedge d\tilde{\Omega} + \frac{k_2}{Q}\left(\frac{k_1}{k_2}\right)^{3/2} \sin(2\theta) dx_{||}\wedge d\Omega\right]. \tag{7.6}$$

Thus the potential felt by a probe $D5$-brane stretched along $(x_0, x_1, x_2, \tilde{S}^3)$ and localized at some ϕ, is attractive

$$\int_{\tilde{S}_2} g_s^{-1}\sqrt{g_{||}g_{\tilde{S}^2}} - A_6 = 4\pi^2 V_{||} e^{Q\phi}\sqrt{2}k_2^{3/2}\left(1-\sqrt{\frac{k_2}{k_1+k_2}}\right). \tag{7.7}$$

The analog of (5.19) for this case implies that the scalar fields are proportional to $\exp(Q\phi/2)$. Thus (7.7) takes the form of a mass term for the scalars. This is in agreement with the field theory analysis of [13] that leads to a mass gap in the dual field theory.

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A. Time dependent supergravity solutions

In section 5 we argued that there are exact CFT's, (5.1) and (5.9), that describe certain time dependent $NS5$-brane configurations. The aim of this appendix is to construct the supergravity solutions associated with these CFT's. Both (5.1) and (5.9) are exactly marginal deformations of the coset $\left(S^1_k \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\right)/\mathbb{Z}_k$. When thinking about the relevant supergravity solutions it is more natural to use the $\mathrm{SU}(2)_k$ description. Therefore, we first review the details of the transformation that takes the $\mathrm{SU}(2)_k$ to $\left(S^1_k \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\right)/\mathbb{Z}_k$. We start on the $\mathrm{SU}(2)_k$ side (in this appendix we set $\alpha' = 1$)

$$ds^2 = k\left(d\theta^2 + \sin^2\theta d\phi_1^2 + \cos^2\theta d\phi_2^2\right), \qquad B_{12} = k\,\cos^2\theta\,, \tag{A.1}$$

where $0 \le \theta \le \pi/2,\ 0 \le \phi_1,\ \phi_2 \le 2\pi$. The complex and Kahler structures associated with this background are

$$\tau = \frac{g_{12}}{g_{22}} + i\frac{\sqrt{g}}{g_{22}} = i\tan\theta\,, \qquad \tau_k = B_{12} + i\sqrt{g} = k\cos\theta e^{i\theta}\,. \tag{A.2}$$

To transform to the $\left(S^1_k \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\right)/\mathbb{Z}_k$ description we first apply T-duality in the ϕ_2 direction. This amounts to $\tau \leftrightarrow \tau_k$ (for a review see [32]) so we find the T-dual background to be

$$ds^2 = k(d\theta^2 + d\phi_1^2) + 2d\phi_1 d\phi_2 + \frac{1}{k\cos^2\theta}d\phi_2^2\,, \qquad B_{12} = 0\,. \tag{A.3}$$

Defining new coordinates

$$\tilde{\phi_1} = \phi_1 + \frac{\phi_2}{k}\,, \qquad \tilde{\phi_2} = \frac{\phi_2}{k}\,, \tag{A.4}$$

we find

$$ds^2 = k\left(d\theta^2 + d\tilde{\phi}_1^2 + \tan^2\theta d\tilde{\phi}_2^2\right). \tag{A.5}$$

Note that now we have a $\mathbb{Z}_k$ identification, $(\tilde{\phi}_1, \tilde{\phi}_2) \sim (\tilde{\phi}_1 + 2\pi/k, \tilde{\phi}_2 + 2\pi/k)$, that implies that the background is indeed $\left(S^1_k \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\right)/\mathbb{Z}_k$. As usual the dilaton picks an extra factor from the T-duality transformation $g_s \to g_s(\det g_{\mathrm{new}}/\det g_{\mathrm{old}})^{1/4}$, so we have

$$g_s \to \frac{g_s}{\cos\theta}\,. \tag{A.6}$$

Now we shall deform the $\left(S^1_k \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\right)/\mathbb{Z}_k$ side in various ways and see what these deformations give on the $\mathrm{SU}(2)_k$ side.

(1) The supersymmetry breaking deformation (4.12) corresponds to changing the radius of the S_1 from $\sqrt{k}$ to $L\sqrt{k}$. The resulting metric on the squashed sphere is

$$ds^2 = k\left(d\theta^2 + L^2 d\tilde{\phi}_1^2 + \tan^2\theta d\tilde{\phi}_2^2\right), \qquad g_s = \frac{1}{\cos\theta}\,. \tag{A.7}$$

The change of coordinates (A.4) leads to

$$ds^2 = k\left(d\theta^2 + L^2 d\phi_1^2 + \frac{2L^2}{k}d\phi_1 d\phi_2 + \frac{L^2 + \tan^2\theta}{k^2}d\phi_2^2\right). \tag{A.8}$$

Applying T-duality in the ϕ_2 direction we get

$$\begin{aligned} ds^2 &= k\left(d\theta^2 + \frac{L^2\tan^2\theta}{L^2+\tan^2\theta}d\phi_1^2 + \frac{1}{L^2+\tan^2\theta}d\phi_2^2\right) \\ B_{12} &= \frac{kL^2}{L^2+\tan^2\theta}\,, \qquad g_s^2 = \frac{1+\tan^2\theta}{L^2+\tan^2\theta}\,. \end{aligned} \tag{A.9}$$

This agrees with the SU(2) contribution to (5.22).

(2) Another useful background is obtained by adding a direction, ρ, and combining it with $\tilde{\phi}_1$ to form a cigar $\left(\frac{\mathrm{SL}(2)_k}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\right)/\mathbb{Z}_k$. This does not change the asymptotic radius of the S^1, so at large ρ the background is $\mathrm{SU}(2)_k \times \mathbb{R}$. The metric is

$$ds^2 = k\left(d\theta^2 + \tan^2\theta d\tilde{\phi}_2^2 + d\rho^2 + \tanh^2\rho d\tilde{\phi}_1^2\right), \qquad g_s = \frac{1}{\cos\theta\cosh\rho}, \qquad (A.10)$$

where $(\tilde{\phi}_1, \tilde{\phi}_2) \sim (\tilde{\phi}_1 + 2\pi/k, \tilde{\phi}_2 + 2\pi/k)$. Now we wish to find the effect of this deformation in the SU(2) variables. Using (A.4) we get

$$ds^2 = k\left(d\theta^2 + d\rho^2 + \tanh^2\rho d\phi_1^2\right) + 2\tanh^2\rho d\phi_1 d\phi_2 + \frac{1}{k}(\tan^2\theta + \tanh^2\rho)d\phi_2^2. \quad (A.11)$$

Applying T-duality we get

$$\begin{aligned} ds^2 &= k\left(d\theta^2 + d\rho^2 + \frac{\tan^2\theta\tanh^2\rho}{\tan^2\theta + \tanh^2\rho}d\phi_1^2 + \frac{1}{\tan^2\theta + \tanh^2\rho}d\phi_2^2,\right) \\ B &= \frac{k\tanh^2\rho}{\tan^2\theta + \tanh^2\rho}, \qquad g_s^2 = \frac{1}{\cos^2\theta\ \cosh^2\rho(\tan^2\theta + \tanh^2\rho)}. \end{aligned} \qquad (A.12)$$

When $\rho \to \infty$ we get back the CHS solution. The full solution corresponds to a ring of fivebranes in the B plane. A simple way to see this is to note that the dilaton diverges at $\theta = \rho = 0$, which is indeed a point in the B plane (see eq. (5.19)).

(3) Combining the backgrounds (1) and (2) leads to $\left(\frac{\mathrm{SL}(2)_{L^2k}}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2)_k}{\mathrm{U}(1)}\right)/\mathbb{Z}_k$. The asymptotic radius is $L\sqrt{k}$ and we also have a cigar. This is the supergravity description of the deformed fivebranes theory perturbed by (5.9). The spacetime fields take the form

$$ds^2 = k\left(L^2(d\rho^2 + \tanh^2\rho d\tilde{\phi}_1^2) + d\theta^2 + \tan^2\theta d\tilde{\phi}_2^2\right), \qquad g_s = \frac{1}{\cos\theta\ \cosh\rho}, \quad (A.13)$$

with the usual $\mathbb{Z}_k$ identifications $(\tilde{\phi}_1, \tilde{\phi}_2) \sim (\tilde{\phi}_1 + 2\pi/k, \tilde{\phi}_2 + 2\pi/k)$. Following the steps above, we first use (A.4) to find

$$ds^2 = kd\theta^2 + L^2kd\rho^2 + L^2k\tanh^2\rho d\phi_1^2 + \frac{1}{k}(L^2\tanh^2\rho + \tan^2\theta)d\phi_2^2 + 2L^2\tanh^2\rho d\phi_1 d\phi_2. \qquad (A.14)$$

Applying T-duality we get

$$\begin{aligned} ds^2 &= k\left(d\theta^2 + L^2d\rho^2 + \frac{L^2\tan^2\theta\tanh^2\rho}{L^2\tanh^2\rho + \tan^2\theta}d\phi_1^2 + \frac{1}{L^2\tanh^2\rho + \tan^2\theta}d\phi_2^2\right), \\ B &= \frac{L^2\tanh^2\rho}{L^2\tanh^2\rho + \tan^2\theta}, \qquad g_s^2 = \frac{1}{\cos^2\theta\cosh^2\rho(L^2\tanh^2\rho + \tan^2\theta)}. \end{aligned} \qquad (A.15)$$

Now we can write down the supergravity solution that corresponds to the deformation (5.9)

$$ds^2 = -dt_{new}^2 + d\phi_{\mathrm{new}}^2 + dx_{||}^2 + k\Bigg(d\theta^2 + \frac{L^2\tan^2\theta\tanh^2(\phi_{\mathrm{new}}/L\sqrt{k})}{L^2\tanh^2(\phi_{\mathrm{new}}/L\sqrt{k}) + \tan^2\theta}d\phi_1^2 +$$

$$+\frac{1}{L^2\tanh^2(\phi_{\rm new}/L\sqrt{k})+\tan^2\theta}d\phi_2^2\Big),$$

$$B = \frac{L^2 k\tanh^2(\phi_{\rm new}/L\sqrt{k})}{L^2\tanh^2(\phi_{\rm new}/L\sqrt{k})+\tan^2\theta},$$

$$g_s^2 = \frac{1}{\cos^2\theta\cosh^2(\phi_{\rm new}/L\sqrt{k})(L^2\tanh^2(\phi_{\rm new}/L\sqrt{k})+\tan^2\theta)\exp(2\alpha t_{\rm new})}, \quad \text{(A.16)}$$

where

$$\alpha^2 = \frac{1}{k}\left(\frac{1}{L^2}-1\right) \quad \text{(A.17)}$$

and $L<1$. A simple way to find the relation between $t_{\rm new}, \phi_{\rm new}$ and the original coordinates t, ϕ is to note that asymptotically the background is not deformed. This gives

$$\phi = \frac{1}{L}\phi_{\rm new}+\sqrt{k}\alpha t_{\rm new}\,, \qquad t = \frac{1}{L}t_{\rm new}+\sqrt{k}\alpha\phi_{\rm new}\,, \quad \text{(A.18)}$$

in agreement with (5.11) .

To verify that this solution indeed describes a ring of $NS5$-branes which run away to infinity we note that the dilaton diverges when $\theta=\phi_{\rm new}=0$. In terms of the original coordinates, the trajectory of the fivebranes is

$$\phi = \left(\frac{1}{L}-L\right)t\,. \quad \text{(A.19)}$$

(4) Now we wish to find the supergravity solution that corresponds to (5.1) . For this we have to proceed in a slightly different way. At large ρ the metric takes the form

$$ds^2 = kd\rho^2+dx^2-dt^2+\cdot \quad \text{(A.20)}$$

where $x\sim x+2\pi R$ and $R=L\sqrt{k}$ with $L<1$. Clearly we can define a new coordinates system obtained by a boost

$$x_{\rm new} = Cx+St\,, \qquad t_{\rm new} = Ct+Sx\,, \qquad C^2-S^2=1 \quad \text{(A.21)}$$

so that the metric at infinity takes the form

$$ds^2 = kd\rho^2+dx_{\rm new}^2-dt_{\rm new}^2+\cdots\,. \quad \text{(A.22)}$$

Now we can deform (A.22) to obtain a cigar like geometry

$$ds^2 = kd\rho^2+\tanh^2\rho\; dx_{\rm new}^2-dt_{\rm new}^2+\cdots \quad \text{(A.23)}$$

Since x is periodic this cannot be done for any boost parameter, C. To avoid a conical singularity we must impose

$$C^2 = \frac{k}{R^2} = \frac{1}{L^2}\,. \quad \text{(A.24)}$$

This condition is equivalent by T-duality to (5.3). Writing (A.23) using t and $\tilde{\phi}_1 = x/R$ we get

$$\begin{aligned} ds^2 &= dx_{||}^2 + k(d\rho^2 + d\theta^2 + \tan^2\theta d\tilde{\phi}_2^2) + d\tilde{\phi}_1^2(k\tanh^2\rho - S^2R^2) - \\ &\quad - dt^2(C^2 - \tanh^2\rho S^2) + 2d\tilde{\phi}_1 dt CSR(\tanh^2\rho - 1)\,. \end{aligned} \tag{A.25}$$

Now we can follow the same steps as above. First we change coordinate from $\tilde{\phi}_i$ to ϕ_i. This gives (with the help of (A.24))

$$\begin{aligned} ds^2 &= dx_{||}^2 + k(d\rho^2 + d\theta^2) + k(\tanh^2\rho - \epsilon)d\tilde{\phi}_1^2 - \frac{1}{L^2}(1 - \epsilon\tanh^2\rho)dt^2 + \\ &\quad + \frac{1}{k}(\tanh^2\rho + \tan^2\theta - \epsilon)d\tilde{\phi}_2^2 + 2(\tanh^2\rho - \epsilon)d\tilde{\phi}_1 d\tilde{\phi}_2 - \\ &\quad - 2\frac{\sqrt{\epsilon k}}{L}\frac{1}{\cosh^2\rho}dt d\tilde{\phi}_1 - \frac{2}{L}\sqrt{\frac{\epsilon}{k}}\frac{1}{\cosh^2\rho}dt d\tilde{\phi}_2\,, \end{aligned} \tag{A.26}$$

where $\epsilon = 1 - L^2$. Applying T-duality in the ϕ_2 direction we get

$$\begin{aligned} \tilde{g}_{22} &= \frac{1}{g_{22}} = \frac{k}{\tanh^2\rho + \tan^2\theta - \epsilon}\,, \\ \tilde{g}_{11} &= \frac{1}{g_{22}}(g_{11}g_{22} - g_{12}^2) = \frac{k\tan^2\theta(\tanh^2\rho - \epsilon)}{\tanh^2\rho + \tan^2\theta - \epsilon}\,, \\ \tilde{g}_{tt} &= \frac{1}{g_{22}}(g_{tt}g_{22} - g_{t2}^2) = -\frac{\tan^2\theta + \tanh^2\rho(L^4 - \epsilon\tan^2\theta)}{L^2(\tanh^2\rho + \tan^2\theta - \epsilon)}\,, \\ \tilde{g}_{t1} &= \frac{1}{g_{22}}(g_{t1}g_{22} - g_{t2}g_{12}) = \sqrt{\frac{\epsilon k}{L^2}}\frac{\tan^2\theta}{\cosh^2\rho(\tanh^2\rho + \tan^2\theta - \epsilon)}\,, \\ \tilde{B}_{t2} &= \frac{g_{t2}}{g_{22}} = -\sqrt{\frac{\epsilon k}{L^2}}\frac{1}{\cosh^2\rho(\tanh^2\rho + \tan^2\theta - \epsilon)}\,, \\ \tilde{B}_{12} &= \frac{g_{12}}{g_{22}} = k\tilde{B}_{t2}\,. \end{aligned} \tag{A.27}$$

This solution describes rotation in the A plane (as opposed to the previous case where the branes move in the B plane) since g_{t2} vanishes and g_{t1} does not.

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